a review of Developments in topological gravity by Dijkgraaf, Robbert; Witten, Edward

Dijkgraaf, Robbert; **Witten, Edward**

Developments in topological gravity. (English) Zbl 1421.81158

Yang, Chen Ning (ed.) et al., Topology and physics. Hackensack, NJ: World Scientific. 17-80 (2019).

There are at least two candidates for quantum gravity in two space-time dimensions, namely, matrix models [*M. R. Douglas*, Phys. Lett., B 238, No. 2–4, 176–180 (1990; Zbl 1332.81211); *D. J. Gross* and *A. A. Migdal*, Phys. Rev. Lett. 64, No. 2, 127–130 (1990; Zbl 1050.81610); *E. Brezin* and *V. Kazakov*, Phys. Lett., B 236, 144–150 (1990)] and topological gravity. It was conjectured by *E. Witten* [Surv. Differ. Geom., Suppl. J. Diff. Geom. 1, 243–310 (1991; Zbl 0757.53049); Nucl. Phys. B 340, 281–332 (1990)] that two dimensional topological gravity is actually equivalent to the matrix model. This equivalence led to formulas for the intersection numbers of certain natural cohomology classes on moduli spaces in terms of the partition function of the matrix model governed by KdV equations [loc. cit., Zbl 1332.81211] or equivalently Virasoro constraints [*R. Dijkgraaf*, *H. L. Verlinde* and *E. P. Verlinde*, Nucl. Phys. B 348, 435–456 (1991)]. This paper, consisting of four sections, aims to provide a good introduction to two developments in topological gravity in this direction.

The formulas for the intersection numbers were first established by *M. Kontsevich* [Commun. Math. Phys. 147, No. 1, 1–23 (1992; Zbl 0756.35081)] by a direct calculation that expressed intersection numbers on moduli space in terms of a new type of matrix model governed again by the KdV and Virasoso constraints. More than a decade later, a new proof of this relationship was found by *M. Mirzakhani* [Invent. Math. 167, No. 1, 179–222 (2007; Zbl 1125.30039); J. Am. Math. Soc. 20, No. 1, 1–23 (2007; Zbl 1120.32008)] by puting the accent on comprehension of the Weil-Petersson volumes of moduli spaces of hyperbolic Riemann surfaces with boundary and showing that these volumes contain all the information in the intersection numbers. §2 gives a glimpse of this story.

Matrix models of two-dimensional gravity are of a natural generalization with vector degrees of freedom added, which is related to two-dimensional gravity on *open* Riemann surfaces. §3 gives some highlights of the take of *R. Pandharipande*, *J. P. Solomon* and *R. J. Tessler* ["Intersection theory on moduli of disks, open KdV and Virasoro", arXiv:1409.2191], which has notched success in establishing an intersection theory on the moduli space of open Riemann surfaces related to matrix models with vector degrees of freedom. §4 shows how matrix models with vector degrees of freedom lead to precisely the same Virasoro constraints that have been found in intersection theory on the moduli space of open Riemann surfaces.

For the entire collection see [Zbl 1414.81013].

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

81V17 Gravitational interaction in quantum theory
81T45 Topological field theories in quantum mechanics
83C80 Analogues of general relativity in lower dimensions
14D21 Applications of vector bundles and moduli spaces in mathematical physics (twistor theory, instantons, quantum field theory)

Full Text: DOI